"DUAL NATURE OF RADIATION AND MATTER".

Worksheet: **MODULE-3**

Q.1] An electron and alpha particle have the same de-Broglie wavelength associated with them. How are their kinetic energies related to each other?

$$E_K = \frac{p^2}{2m} \qquad \text{where} \left[\begin{array}{l} E_K = \text{Kinetic energy} \\ p = \text{momentum} \\ m = \text{mass of the particle} \end{array}\right.$$

de-Broglie wavelength, $\lambda = \dfrac{h}{p}$

...where [h = Planck's constant

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE_K}}$$

$\because$ Both the particles have the same de-Broglie wavelength ...[Given

$$\therefore \quad \frac{h}{\sqrt{2m_e E_{Ke}}} = \frac{h}{\sqrt{2m_\alpha E_{K\alpha}}}$$

$$\text{or} \quad \frac{m_e}{m_\alpha} = \frac{E_{K\alpha}}{E_{Ke}} \qquad \text{where} \left[\begin{array}{l} m_e = \text{mass of electron} \\ m_\alpha = \text{mass of } \alpha\text{ - particle} \\ E_{Ke} = \text{K.E. of electron} \\ E_{K\alpha} = \text{K.E. of } \alpha\text{ - particle} \end{array}\right.$$

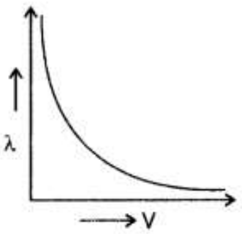
As $m_\alpha > m_e$ $\qquad \therefore$ K.$E_{Ke} > E_{K\alpha}$

Q.2] Show graphically, the variation of the de- Broglie wavelength (λ) with the potential (V) through which an electron is accelerated from rest.

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}}$$

$$= \frac{h}{\sqrt{2meV}}$$

$$= \frac{12.27}{\sqrt{V}} \text{Å}$$



Q.3] State de-Broglie hypothesis.

According to de Broglie, a moving material particle can be associated with a wave. i.e. a wave can guide the motion of the particle. The waves associated with the moving material particles are known as de Broglie waves or matter waves.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Q.4] A proton and an electron have same kinetic energy. Which one has smaller de-Broglie wavelength and why?

K.E. of a particle, $K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{m}$

$\therefore$ Linear momentum, $p = \sqrt{2mK}$

de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

For the particles possessing same kinetic energy,

$\lambda \propto \frac{1}{\sqrt{m}}$

$me << mp \quad \therefore \lambda_e >> mp \quad \therefore \lambda_e >> h_p$

Proton has smaller de-Broglie wavelength.

Q.5] Write the expression for the de Broglie wavelength associated with a charged particle having charge 'q' and mass 'm', when it is accelerated by a potential V.

$$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$$

Q.6] Find the ratio of de-Broglie wavelengths associated with two electrons accelerated through 25 V and 36 V.

$\lambda \propto \frac{1}{\sqrt{V}}$ $\qquad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$

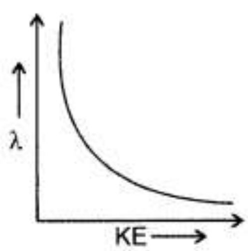
$\therefore \lambda_1 : \lambda_2 :: \mathbf{6 : 5}$

Q.7] An electron is revolving around the nucleus with a constant speed of 2.2 × 108 m/s. Find the de-Broglie wavelength associated with it.

de-Broglie wavelength $(\lambda) = \frac{h}{mv}$

$$= \frac{6.63\times10^{-34}}{(9.1\times10^{-31})\times(2.2\times10^{8})} = \mathbf{3.31 \times 10^{-12}\ m}$$

Q.8] Draw a plot showing the variation of de Broglie wavelength of electron as a function of its K.E.



Q.9] An electron is accelerated through a potential difference of 64 volts. What is the de-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond?

According to de-Broglie wavelength,

$$\lambda = \frac{1.227}{\sqrt{V}}\text{ nm} = \frac{1.227}{\sqrt{64}} = \frac{1.227}{8} = 0.1533\text{ nm}$$

This wavelength is associated with X-rays.

Q.10] An a-particle and a proton are accelerated from rest by the same potential. Find the ratio of their de-Broglie wavelengths.

Answer:

de-Broglie wavelength of a charged (q)

Particle accelerated through a potential ‘V’ is

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Ratio of their de-Broglie wavelengths for an α-particle and a proton is $\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{q_p m_p}{q_\alpha m_\alpha}}$

As $q_\alpha = 2\,q_p,\ m_\alpha = 4\,m_p$

$$\therefore \quad \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)} = \frac{1}{2\sqrt{2}}.$$

Q.11] Plot a graph showing the variation of stopping potential with the frequency of incident radiation for two different photosensitive materials having work functions W1 and W2 (W1 > W2). On what factors does the

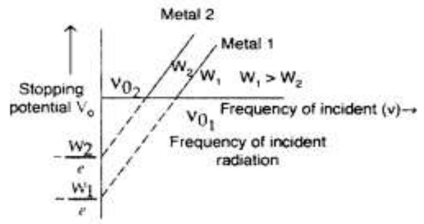
(i) slope and

(ii) intercept of the lines depends?

(*i*) As $eV_0 = h\upsilon - W_0 \qquad V_0 = \left(\frac{h}{e}\right)\upsilon - \frac{W_0}{e}$

$\therefore$ Slope of $V_0 - \upsilon$ graph $= \frac{h}{e}$

Slope depends on the Planck's constant and electronic charge (*e*).



(*ii*) Intercept $= \frac{\text{Work function}}{e} = \frac{-W_0}{e}$

Therefore, Intercept depends upon the work function (threshold frequency) and electronic charge (*e*).

Q.12] A proton and a deuteron are accelerated through the same accelerating potential. Which one of the two has?

(a) greater value of de-Broglie wavelength associated with it, and

(b) less momentum?

Give reasons to justify your answer.

For proton and deuteron, charge (q) is the same, while the mass of deuteron is more than that of proton

(a) We know $\lambda = \frac{h}{\sqrt{2mqV}}$

Here q and V are the same for both,

Hence, $\lambda \propto \frac{1}{\sqrt{m}}$

∴ Proton will be associated with greater value of de-Broglie wavelength.

(b) $\because \frac{1}{2}mv^2 = qV$

Multiplying 'm' on both sides, we have

$\frac{1}{2}m^2v^2 = mqV$ or $\frac{1}{2}p^2 = mqV$

or $p = \sqrt{2mqV}$ $\therefore p \propto \sqrt{m}$

Proton will have less momentum.

Q.13] A proton and an alpha particle are accelerated through the same potential. Which one of the two has?

(i) greater value of de-Broglie wavelength associated with it, and

(ii) less kinetic energy.

Give reasons to justify your answer.

Similar to Q. 12.

[Hint. Proton's mass is less than that of alpha particle, which contains 2 protons and 2 neutrons.]

Q.14] A deuteron and an alpha particle are accelerated with the same accelerating potential. Which one of the two has?

(1) greater value of de-Broglie wavelength, associated with it, and

(2) less kinetic energy? Explain.

Similar to Q. 12.

[Hint. A deuteron (consisting of one proton and one neutron) has less mass than alpha particle (consisting of 2 protons and 2 neutrons)]

Q.15] (i) Monochromatic light of frequency 6.0 × 1014 Hz is produced by a laser. The power emitted is 2.0 × 10-3 W. Estimate the number of photons emitted per second on an average by the source.

(ii) Draw a plot showing the variation of photoelectric current versus the intensity of incident radiation on a given photosensitive surface.

(i) Given : $\nu = 6.0 \times 10^{14}$ Hz, $P = 2.0 \times 10^{-3}$ W

Energy of one photon $= h\nu$

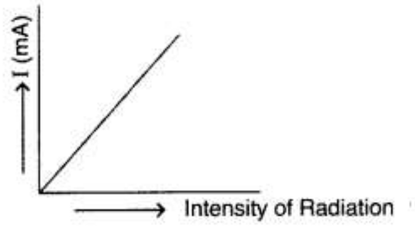
$= (6.6 \times 10^{-34}) \times (6.0 \times 10^{14})$

Number of photons emitted per sec

$$= \frac{\text{Power}}{\text{Energy of one photon}}$$

$$n = \frac{2\times10^{-3}}{(6.6\times10^{-34})\times(6.0\times10^{14})} \quad \therefore \mathbf{n = 5 \times 10^{15}}$$

(ii)



Q.16] Derive the Bohr's quantisation condition for angular momentum of the orbiting of electron in hydrogen atom, using de Broglie's hypothesis.

By de Broglie's hypothesis

We have $\lambda = \frac{h}{p} = \frac{h}{mv_n}$

From Bohr's postulate,

$$2\pi r_n = n \qquad (n = 1, 2, 3)$$

$$\therefore \quad 2\pi r_n = n\left(\frac{h}{mv_n}\right)$$

$$mv_n r_n = \frac{nh}{2\pi}$$

$$\therefore \quad \text{Angular mometum} = (mv_n r_n) = \frac{nh}{2\pi}$$

Q.17] State two properties of photons. For a monochromatic radiation incident on a photosensitive surface, why do all photoelectrons not come out with the same energy? Give reason for your answer.

Two properties of photons: i) photon is electrically neutral. ii) photon has an energy equal to hv

For a monochromatic radiation incident on a photosensitive surface, all photoelectrons do not come out with the same energy, because in addition to the work done to free electrons from the surface, different (emitted) photoelectrons need different amount of work to be done on them to reach the surface.